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Tuning Multi Criteria Evolutionary Algorithms for Airfoil Design Optimization

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Abstract

Statistical design of experiments methods are used to improve the parameterization of a multi-objective evolution strategy on an airfoil design problem. The application of these methods require a measure to compare the approximated Pareto fronts generated by a multi-objective evolution strategy. A new technique is proposed that determines intersection points of attainment surfaces with cross-lines. The parameterization of the evolution strategy and consequently the results of the airfoil design test-case under investigation can be improved.

1 Introduction

This article deals with evolution strategies (ES) that build a special class of evolutionary algorithms (EA), see [BS02] for a comprehensive introduction. We will investigate different ES parameter settings using DOE techniques. This will lead to results, that are tailored for each specific optimization problem. Nevertheless, the methodology presented in this paper is applicable to any kind of parameterizable search algorithm such as simulated annealing, tabu search or genetic algorithms.

Stochastic search algorithms such as evolutionary algorithms are considered as alternatives to classical search and optimization techniques. The latter generate one single optimized solution in one run whereas the former, as population based algorithms, can produce a set of solutions. Optimization runs require the specification of exogenous algorithm parameters such as the population size or the selective pressure. Since many real-world problems are computationally expensive, only few optimization runs are possible, that should be performed with "good" exogenous parameters. We will consider the naive, but intuitively understandable characterization of "good" strategy parameters: they lead to robust solutions, that are found with a minimum amount of fitness function evaluations. The reader is referred to [BB03] or [ES03] for a more detailed discussion of this topic. Design of Experiments (DOE) techniques can be applied to optimization algorithms, considering the run of an algorithm as an experiment, gaining knowledge about the behavior of the algorithm and the interactions and significances of its parameters. To profit from the chances DOE techniques offer to improve optimization algorithms, the performances of these algorithms using different parameterizations have to be compared.

In the current case of a multi-objective optimization task, the outcome of an algorithm run is a set of points that build an approximation of a Pareto optimal front. The resulting solutions should be close to and diversely distributed over the Pareto optimal front [Deb01]. To compare these approximated Pareto fronts, different techniques have been suggested using lines crossing the associated attainment surfaces [FF96, KC02]. Here, the approach from [NWBH02] is extended by using parallel lines in the corresponding sector of the search space. The sections of the y-axes can easily be used to distinguish between the parallels and to improve the density of lines in this approach. The sequence of sections from attainment surfaces of different experiments and the cross-lines are used to measure the performance of the corresponding parameterization.

The problem under investigation is the design of the profile of an aircraft wing. The design of whole aircrafts is a complex and very expensive task. It requires the fulfillment of different objectives and exhibits multiple domains for optimization. One of these domains is the aircraft wing design. The task is to optimize the profile of a wing, an airfoil, given optimized airfoils for two different flight conditions. Therefore, the airfoil optimization problem belongs to the class of multi-objective optimization problems.

The rest of the paper is organized as follows: section 2 gives an introduction to airfoil design problems. The ES and its parameterization is briefly described in section 3. In addition, DOE and regression basics that are used to perform a comparison of different optimization run configurations are presented in this section. An approach that extracts a set of points from attainment surfaces generated from different optimization runs to enable a statistical comparison is introduced in section 4. A constructive example and experimental results are presented in section 5. Finally, section 6 gives a summary.

2 Airfoil Design Optimization

To avoid the tradeoff between computational expensive highly sophisticated computational fluid dynamics (CFD) methods and reduced preciseness and reliability of results in aircraft design, an aircraft is subdivided into logical parts for optimization. In this paper only the wing is studied. But

Table 1: Summarized design conditions (c=chord length)

Property	Case	high lift	low drag
M_∞	$[-]$	0.20	0.77
Re_c	$[-]$	$5 \cdot 10^6$	10^7
α	$[^0]$	10.8	1.0

concentrating on this 3-dimensional parts is still too hard for todays computational resources to solve with the necessary preciseness in times allowing for the coupling to optimization procedures. Instead we decided to tackle the 2-dimensional airfoil design problem under different design conditions. In reality, the design conditions given by the Mach- and Reynolds-number, angle of attack, etc. change continuously. In our test case two design conditions have been fixed, one describing a sub-sonic (high-lift) condition, the other one describing a transonic (low-drag) condition. The exact design conditions are shown in table 1. The task in this special re-design test case is to minimize the difference in the pressure distribution between the airfoil proposed by the optimization procedure and the target airfoils given for each design condition. The intention here is to identify the best optimization procedures for general airfoil design problems or even design problems in general. Therefore the objective functions read:

$$F_i(s) = \int_0^1 (C_p(s) - C_{p,target_i}(s))^2 ds \quad i \in \{1, 2\},$$

with s being the airfoil arc-length measured around the airfoil. C_p is the pressure coefficient distribution of the current and $C_{p,target_{1,2}}$ the pressure coefficient distribution of the target airfoils, respectively.

The multi-point test case with this design conditions has been under investigation before with a multi-objective $(1+10)-$ES using de-randomized step size adaption (MODES III). This algorithm selects a non-dominated solution with the largest difference in solutions space to other non-dominated solution for becoming the parent of the next generation [NWBH02]. In this article we consider multi-membered ES with μ parents individuals. The parents are collected by executing the above algorithm μ times.

3 Evolution Strategies and Experimental Designs

Evolution strategies are population based, randomized search heuristics. They can be described briefly as follows: a parent population P of size μ is initialized at time $t = 0$. Then λ offspring individuals are generated by selecting a parent family of size ρ from P. Recombination and mutation are

Table 2: Exogenous parameters of an evolution strategy.

Symbol	Factor	Parameter	Range	Typical Values
μ	P	Number of parent individuals	$\mathbb{N}$	15
$\nu = \lambda/\mu$	S	Offspring-parent ratio	$\mathbb{R}_+$	7
$\sigma_i^{(0)}$	I	Initial standard deviations	$\mathbb{R}_+$	3
n_σ	N	Number of standard deviations. D denotes the problem dimension	$\{1, D\}$	D
c_τ	T	Multiplier for individual and global mutation parameters	$\mathbb{R}_+$	1
ρ	R	Mixing number	$\{1, \mu\}$	μ
r_x	X	Recombination operator for object variables	$\{i, d\}$	d (discrete)
r_σ	S	Recombination operator for strategy variables	$\{i, d\}$	i (intermediate)
κ	K	Maximum age	$\mathbb{R}_+$	1

applied to the offspring individuals and a selection is performed to determine the next parent population. This process continues with the generation of the next set of offspring individuals until a termination criterion is fulfilled. The reader is referred to [BS02] for an comprehensive introduction to ES.

The exogenous parameters that have to be determined before an ES run is started are summarized in table 2. Fine–tuning of ES parameters might improve the algorithm and reveal information about its robustness. The role of the parent–offspring ratio $\nu = \lambda/\mu$, or the relationship between recombination and mutation operator might be important to improve the behavior of the algorithm. Thus, we interpret an optimization run as an experiment. The experimenter can modify the exogenous parameters. Experimental design provides an excellent way of deciding which optimization runs should be performed so that the desired information can be obtained with the least amount of experiments [Mon01]. The input parameters and structural assumptions, that define a optimization algorithm are called factors, the output value(s) are called response(s). The different values of parameters are called levels. Levels can be qualitative, i.e. selection scheme, or quantitative, i.e. population size. An experimental design is a set of factor level combinations. One parameter design setting is run for different pseudo-random number settings, resulting in replicated outputs. Kleijnen classifies the commonly used "one factor at a time approach" to enhance the behavior of the algorithm as inefficient and ineffective and recommends DOE [KG92].

Table 3: Fractional factorial 2_{III}^{5-2} design.

i	P	O	K	X	S	P	O	K	X	S	y_1	y_2	y_3	y_4	y_5
1	−	−	−	+	+	2	2	1	i	i	.0	.07	.0	.07	.06
2	+	−	−	−	−	5	2	1	d	d	.06	.0	.07	.0	.09
3	−	+	−	−	+	2	5	1	d	i	.0	.0	.0	.16	.0
4	+	+	−	+	−	5	5	1	i	d	.18	.35	.67	.14	.03
5	−	−	+	+	−	2	2	250	i	d	.09	.0	.19	.08	.01
6	+	−	+	−	+	5	2	250	d	i	.11	.17	.02	.1	.15
7	−	+	+	−	−	2	5	250	d	d	.18	.0	.04	.21	.45
8	+	+	+	−	+	5	5	250	d	i	.38	.41	.01	.24	.21

Generally, a optimization model can be represented as follows:

$$y = f_1(z_1, \ldots, z_k, r_0), \tag{1}$$

where f_1 is a mathematical function, e.g. $f_1 : \mathbb{R}^{k+1} \to \mathbb{R}$: Given the values of the argument z_i and the random number seed r_0, the optimization program yields exactly one value. The Taylor series expansion yields the first order approximation $y = f_2 = \sum_{i=0}^{k} \beta_i z_k$. The last equation is the basis for regression models based on optimization data. Our goal is to use least square methods to estimate the linear model $y = X\beta + \epsilon$, where the y denotes a column vector with the n responses, ϵ is the vector of n error terms, and β denotes the vector with q parameters β_j ($n \geq q$). We will consider orthogonal designs with two factors for each level (2^k factorial designs), and orthogonal design where center points are added to the 2^k design (central composite designs). A variable x is called standardized, if x ranges between -1 and $+1$. The original variables with range $[l, h]$ can be standardized using the linear transformation $x = a + bz$ with $a = (l+h)/2$ and $b = (l-h)/2$. Thus, the entry -1 in the design matrix denotes a factor at its lowest level, and $+1$ a factor at its highest level. Table 3 shows the parameter settings used in this study: as the number of fitness function evaluations is restricted, only small population sizes appear to be reasonable.

The two levels of factor P were chosen as 2 and 5 (consult table 2 for a mapping of EA parameters to factors). The same values have been selected for the two levels of the selective pressure. Hence, the following parent-offspring population sizes have been used for the first design: (2+4), (2+10), $(5 + 10)$, and $5 + 25$. The maximum age in generations of the individuals K was varied from 1 to 250. Two different recombination schemes, global discrete and global intermediate have been selected for the object and strategy parameters (factors X and S), respectively. Experiments based on this design produced the fitness values y_i shown in the last five columns of table 3. Section 4 describes how these values have been determined. Response surface methods provide means to determine the direction of improvement using the path of the steepest descent (minimization problem) based on the estimated first-order model [KG92].

4 Comparing the Performance of Different Multi-Objective Evolutionary Algorithms

Attainment surfaces mark all solutions that are sure to be dominated by the set of already obtained non-dominated solutions. The objective space is divided into two regions whether the points are dominated by the results of the algorithm or not [FF96, KC00, KC02]. Repeated runs of the multi-objective evolutionary algorithm (MOEA) result in a set of attainment surfaces $\mathcal{A}_i = \{A_{ij}\}$ for the ith run configuration $(i = 1, \ldots, m; j = 1, \ldots, k_i)$, if the ith run configuration is repeated k_i times. Intersecting attainment surfaces with cross-lines enable us to define a metric for a comparison of several MOEA parameter design configurations. Cross-lines can be defined as

1. diagonal imaginary lines running in the direction of the improvement in all objectives [FP96],
2. lines intersecting the origin [KC00], or
3. lines that are parallel to the first bisector of the angle [NWBH02]. The forthcoming investigations are based on this approach.

Eight different parameter design configurations are compared, $(i = 1, \ldots, 8)$. The jth run, $j = 1, \ldots, 5$, of the ith algorithm parameter configuration defines an attainment surface A_{ij} and gives the point of intersection S_{ijk} with the kth cross-line L_k. Figure 1 visualizes this situation with three cross-lines and one attainment surface. Thus, for every run configuration we obtain a distribution of points of intersections on every cross-line, see figure 2. Finally, the percentage of the cross-lines on which the ith run configuration performs best is determined. This value induces a ranking on the set of run configurations under consideration.

As the cross-lines can be chosen arbitrarily, the optimization practitioner can easily define regions of special interest in the objective space. Figure 3 reveals that already a small number of cross-lines can give a good approximation of the quality of an algorithms parameterization.

5 Results

In the following, DOE methods and the cross-line ranking technique are combined to improve the ES algorithm. We are also interested in a robust

ES parameterization that can be found systematically and efficiently.

Starting point of this experiment is the analysis of the parameterization of the ES on the airfoil-design optimization problem as shown in table 1. The maximum number of fitness function evaluations was set to 1000. In the first

Table 4: ES parameter designs for the 18 dimensional airfoil-design.

ES	Airfoil-Design Model ($D = 18$)		
Variable	First design Low (-1)	Up ($+1$)	Final value
$z_1 = P$	2	5	6
$z_2 = O$	2	5	7
$z_3 = K$	1	250	125
$z_4 = X$	i	d	i
$z_5 = S$	i	d	d

step of our analysis we are only interested in the influence of the main effects on the optimization process. Therefore we chose a 2^{5-2}_{III} fractional factorial design shown in table 3, that requires only eight different parameter settings. The first design is shown in table 4. Standardizing the original variables and applying techniques from regression analysis reveals that the population size (factor P) and the selective pressure (factor O) are significant.[1]

For both factors higher values improve the algorithm's performance. This can also be seen from the regression equation[2]

$$\begin{aligned}\hat{y} = &- 0.1954341 + 0.0373030x_1 + 0.0463030x_2 \\ &+ 0.0003129x_3 + 0.0458182x_4 - 0.0055455x_5,\end{aligned} \tag{2}$$

and is depicted in figure 4.

Based on equation 5, the (normalized) direction of the steepest descent can be determined. As μ (factor P) and ρ (factor O) are statistically significant, we can restrict the line search to the related two dimensions. The other settings are held constant during the line-search. Starting from the center point $Z_0 = (4, 4, 125, i, d)$, we can can generate the following points to perform the line search:

$$Z_{01} = (4.2672, 4.5, 125, d, i) \ ,$$

[1]Significance is used here in the sense of classical definitions from statistics.

[2]The x_i denote the standardized variables, whereas z_i denote the corresponding natural variables. The regression analysis is performed on the standardized variables [Mon01].

$$Z_{02} = (5.0345, 5.5, 125, d, i) , \ldots$$

To determine the population size, the values from the first two entries have to be rounded to the next whole integer, since we can perform the optimization with whole individuals only. A comparison of the approximated Pareto optimal fronts from the first first design to the improved design is shown in Figure 5.

6 Summary

A performance measure that enables the application of statistical design of experiment methods to multi-criteria optimization algorithms was proposed. Different ES parameter settings for a an airfoil optimization task were compared using DOE to improve the algorithm's performance. We extended the methods used in [NWBH02], that are based on Fonseca's and Fleming's attainment-surface approach [FP96]. The approach presented in this article is a flexible. Specific user preference can be easily integrated, i.e. by specifying additional cross-lines in the region of interest. Furthermore, it is intuitively understandable and can be combined with other statistical methods. The experimental results show that DOE methods provide convenient means to improve the performance of MOEA, especially if the evaluation of the fitness function is very costly. Therefore we recommend to perform a few tests (with parameter settings based on fractional factorial designs) to determine a "good" strategy parameter setting for the real optimization. This preliminary analysis can easily be accomplished with statistical software packages such as R [IG96].

References

[BB03] T. Bartz-Beielstein. Experimental analysis of evolution strategies - overview and comprehensive introduction. Technical Report of the Collaborative Research Center 531 *Computational Intelligence* CI–157/03, University of Dortmund, 2003.

[BS02] H.-G. Beyer and H.-P. Schwefel. Evolution strategies – A comprehensive introduction. *Natural Computing*, 1:3–52, 2002.

[Deb01] K. Deb. *Multi-Objective Optimization using Evolutionary Algorithms.* Wiley, New York, 2001.

[ES03] A.E. Eiben and J.E. Smith. *Introduction to Evolutionary Computing.* Springer, 2003.

[FF96] C.M. Fonseca and P.J. Fleming. On the performance assessment and comparison of stochastic multiobjective optimizers. In H.-M.

Voigt, W. Ebeling, I. Rechenberg, and H.-P. Schwefel, editors, *Parallel Problem Solving from Nature IV*, pages 548–593, 1996.

[FP96] C.M. Fonseca and P.J.Fleming. On the performance assement and comparison of stochastik multiobjective optimizers. In H.-M. Voigt, W. Ebeling, I. Rechenberg, and H.-P. Schwefel, editors, *PPSN*, volume 1141 of *Lecture Notes in Computer Science*. Springer, 1996.

[IG96] R. Ihaka and R. Gentleman. R: A language for data analysis and graphics. *Journal of Computational and Graphical Statistics*, 5(3):299–314, 1996.

[KC00] J. D. Knowles and D. Corne. Approximating the nondominated front using the pareto archived evolution strategy. *Evolutionary Computation*, 8(2):149–172, 2000.

[KC02] J. Knowles and D. Corne. On metrics for comparing non-dominated sets. In D. B. Fogel, M. A. El-Sharkawi, X. Yao, G. Greenwood, H. Iba, P. Marrow, and M. Shackleton, editors, *Proceedings of the 2002 Congress on Evolutionary Computation CEC2002*, pages 777–782. IEEE Press, 2002.

[KG92] J. P. C. Kleijnen and W. Van Groenendaal. *Simulation - A Statistical Perspective*. Wiley, Chichester, 1992.

[Mon01] D. C. Montgomery. *Design and Analysis of Experiments*. John Wiley & Sons, New York, NY, 5 edition, 2001.

[NWBH02] B. Naujoks, L. Willmes, T. Bäck, and W. Haase. Evaluating multi-criteria evolutionary algorithms for airfoil optimisation. In J. J. Merelo Guervós et al., editor, *Parallel Problem Solving from Nature – PPSN VII, Proc. Seventh Int'l Conf., Granada, September 2002*, pages 841–850, Berlin, 2002. Springer.

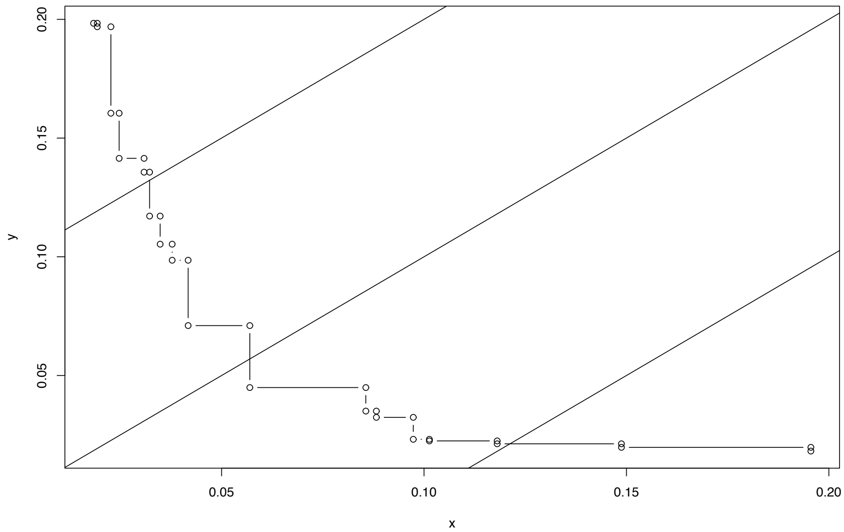

Figure 1: Attainment surface with cross-lines for one run of one simulation run-configuration. Low-drag is plotted against high-lift.

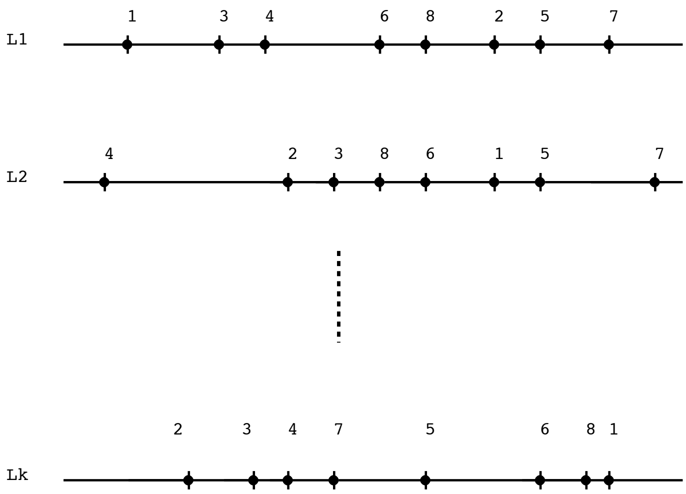

Figure 2: Points of intersection on cross-lines. The first run configuration performs best on cross-line L_1, whereas the fourth run configuration performs best on L_2.

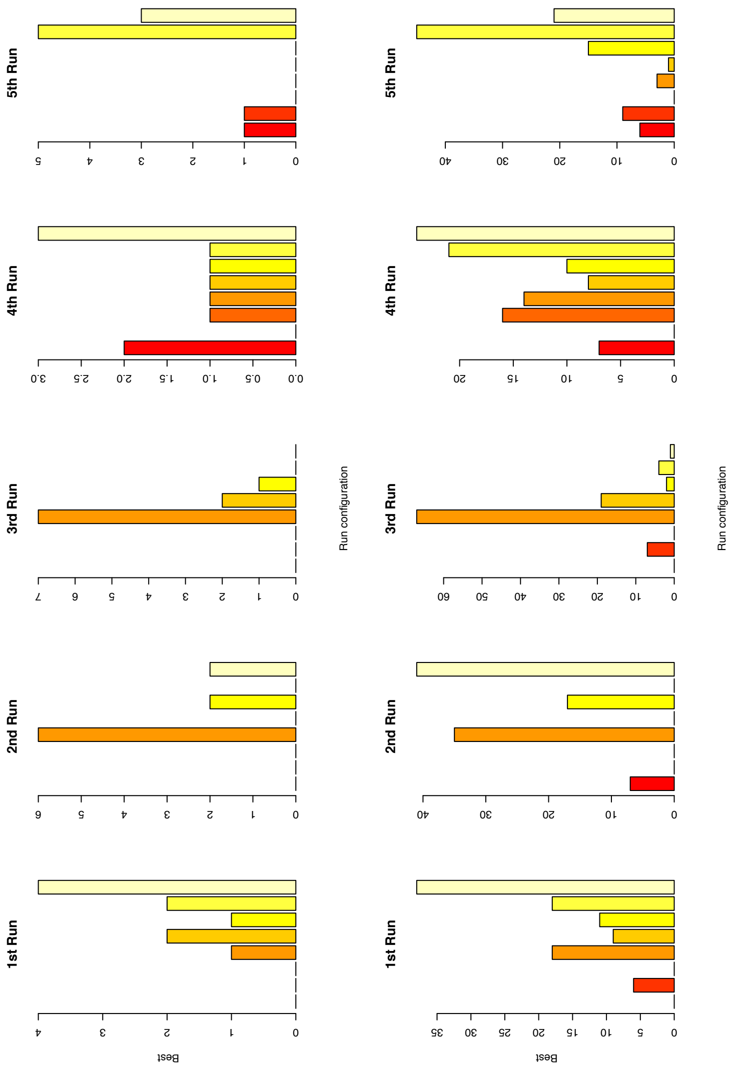

Figure 3: Barplots of the distributions of the best parameter design configurations. The eight different run-configurations from table 3 are compared. The 1st row from above shows the results of a comparison based on 10 crosslines, whereas the 2nd row from above shows the same comparison based on 100 crossing lines. These plots reveal that already a small number of crossing lines can give a good approximation of the quality of an parameter design configuration.

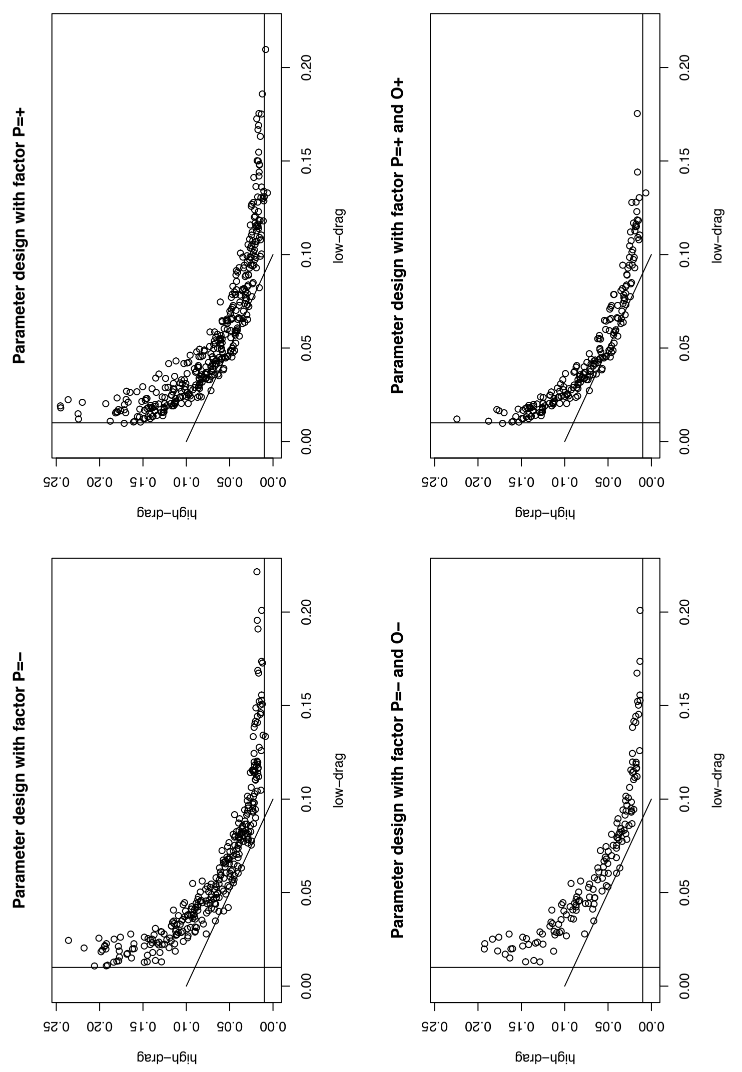

Figure 4: Comparison of four different factor settings, see table 3. The first figure in the first row from above shows the results if the factor P (population size) is set to a low value (here: $\mu = 2$), whereas the second figure in the first shows the situation, when P was set to a high value (here: $\mu = 5$). The first figure in the second row shows the results if both factors are set to a low value, whereas the second figure display the results if both factors are set to its high values. The identical line segments are included in the figures to enhance the comparability of the results.

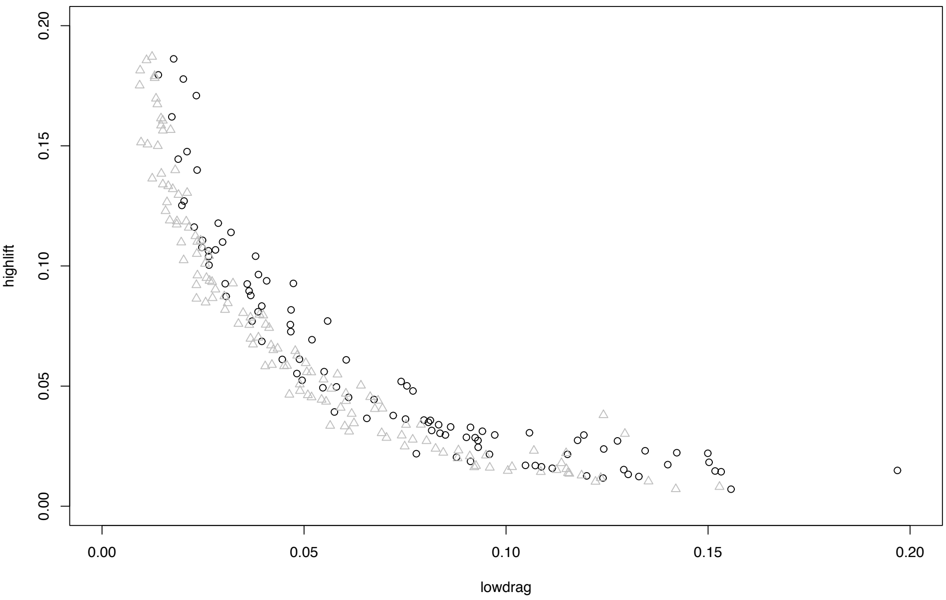

Figure 5: Comparison of the first and the improved factor setting, see table 4. Black circles denote the results from the initial design, whereas grey triangle denote the results from the improved design. Both objectives have to be minimized.